Angular Motion in a Plane – cont'd

Week 9, Lesson 1

- Centripetal Acceleration
- Centripetal Force
- Newton's Law of Gravitation

References/Reading Preparation:

Schaum's Outline Ch. 9

Principles of Physics by Beuche – Ch.7

Centripetal Acceleration

A point mass *m* moving with constant speed *v* around a circle of radius *r* is undergoing acceleration.

This is because although the magnitude of its linear velocity is not changing, the ***direction*** of the velocity is continually changing.

This change in velocity gives rise to an acceleration a_c of the mass – recall that acceleration $\bar{\mathbf{a}}$ = change in velocity / time taken.

This acceleration is directed toward the centre of the circle, and is called the ***centripetal acceleration.***.

The value of the ***centripetal acceleration*** is given by:

$$a_c = \frac{(\text{tangential speed})^2}{\text{radius of circular path}}$$

$$= \frac{v^2}{r}$$

Where: v = speed of the mass around the circular path

r = radius of the circular path

Because $v = \omega r$, we also have $a_c = \omega^2 r$

where ω is measured in radians.

Centripetal Force

Newton’s first law states that a net force must act on an object if the object is to be deflected from straight-line motion.

Therefore an object traveling on a circular path must have a net force deflecting it from straight-line motion.

For example, a ball being twirled in a circular path is compelled to follow this path by the center-ward pull of the string.

If the string breaks, then the ball will follow a straight line path tangent to the circular path from the point at which it is released.

Now that we know about the centripetal acceleration that acts towards the centre of the circular path, computing the force needed to hold an object of mass *m* in a circular path is a simple task.

We now know that $a_c = v^2/r$ (centripetal acceleration directed towards the centre of the circular path)

Now, a force in the same direction, toward the centre of the circle, must pull on the object to furnish this acceleration.

From the equation $\mathbf{F}_{\mathrm{net}} = m\boldsymbol{a}$, we find this required force.

$$F_c = ma_c = \frac{mv^2}{r}$$

Where *Fc* is called the ***centripetal force***, and is directed toward the centre of the circle.

Worked Example

A 1200 kg car is turning a corner at 8.00 m/s, and it travels along an arc of a circle in the process.

a) If the radius of the circle is 9.00 m, how large a horizontal force must the pavement exert on the tires to hold the car in a circular path?
b) What minimum coefficient of friction must exist in order for the car not to slip?

(ans. 8530 N, 0.725)

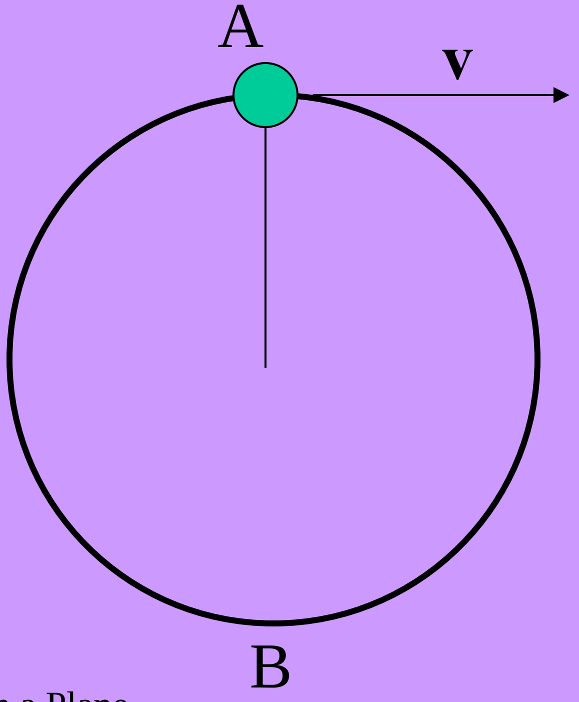
Worked Example

A mass of 1.5 kg moves in a circle of radius 25 cm at 2 rev/s. Calculate a) the tangential velocity, b) the centripetal acceleration, and c) the required centripetal force for the motion.

(ans. 3.14 m/s, 39.4 m/s^2 radially inward, 59N)

Worked Example

A ball tied to the end of a string is swung in a vertical circle of radius *r*. What is the tension in the string when the ball is at point A if the ball's speed is *v* at that point? (Do not neglect gravity). What would the tension in the string be at the bottom of the circle if the ball was going speed *v*?



Worked Example

A curve in a road has a 60 m radius. It is to be banked so that no friction force is required for a car going at 25 m/s to safely make the curve. At what angle should it be banked?

(ans. 47°)

Newton's Law of Gravitation

Two uniform spheres with masses m_1 and m_2 that have a distance r between centres attract each other with a radial force of magnitude:

$$F = G\,\frac{m_1 m_2}{r^2}$$

where G = gravitational constant = $6.672 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$

Illustration

Two uniform spheres, both of 70.0 kg mass, hang as pendulums so that their centres of mass are 2.00 m apart. Find the gravitational force of attraction between them.

(ans. 8.17×10^{-8} N)

Example

A spaceship orbits the moon at a height of 20,000 m. Assuming it to be subject only to the gravitational pull of the moon, find its speed and the time it takes for one orbit. For the moon, m_m = 7.34 x 10^{22} kg, and r = 1.738 x 10^{6} m.

(ans. 1.67 km/s, 110 min)